What is a Matrix

1. Definition

A rectangular arrangement of elements is called a **matrix**:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$$

The elements of a matrix are usually numbers. However, variables and functions can also occur.

The position of an element — e.g. a_{ij} — is indicated by a double index:
the first index i gives the row and the second index j gives the column in which the element is located.

A matrix consisting of m rows and n columns is called an (m, n)-matrix.

Example 1

$$A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \\ 6 & 5 \end{pmatrix}$$

The matrix A is a $(3, 2)$-matrix.

Example 2

$$B = \begin{pmatrix} 2 & 1 & -3 \\ 5 & -7 & 6 \end{pmatrix}$$

The matrix B is a $(2, 3)$-matrix.

A matrix consisting of m rows and n columns has the dimension $m \times n$.

2. Calculating with Matrices

Matrices can be added, subtracted, and multiplied. In addition, matrices can be transposed and inverted. How this works and what you need to pay attention to is explained in the following chapters:

- Adding matrices / subtracting matrices
- Multiplying matrices
- Transposing matrices
- Inverting matrices

Operation	Requirement
Add matrices	The number of rows and columns of A and B must match
Subtract matrices	The number of rows and columns of A and B must match
Multiply matrices	The number of columns of A must equal the number of rows of B

Division of matrices is not defined. In some cases, however, multiplication by the inverse matrix is possible:

$$A/B = A \cdot B^{-1}$$

3. Special Matrices

In the following, some matrices are presented that differ from other matrices due to their special structure.

3.1. Square Matrices

A matrix whose number of rows and columns is the same $(m = n)$ is called **square**.

Well-known representatives of this type are the 2×2 and 3×3 matrices, which frequently appear in school and university studies.

Example 5

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

The elements of a square matrix for which $i = j$ form the so-called **main diagonal** of the matrix

3.2. Zero Matrix

A matrix whose elements are all equal to zero is called a **zero matrix.**

Example 6

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Example 7

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

3.3. Identity Matrix

A matrix in which the elements of the main diagonal are equal to one and all other elements are equal tc zero is called an **identity matrix**.

Example 8

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Example 9

$$B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3.4. Diagonal Matrix

A matrix in which all elements — except for the elements on the main diagonal — are equal to zero is called a **diagonal matrix**.

Example 10

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Special cases

- **Identity matrix** (elements of the main diagonal are equal to one)
- **Zero matrix** (elements of the main diagonal are equal to zero)

3.5. Upper Triangular Matrix

A matrix in which all elements below the main diagonal are equal to zero is called an **upper triangular matrix.**

Example 11

$$A = \begin{pmatrix} 3 & 4 & 1 \\ 0 & -5 & 4 \\ 0 & 0 & 4 \end{pmatrix}$$

Special case

- **Zero matrix**

3.6. Lower Triangular Matrix

A matrix in which all elements above the main diagonal are equal to zero is called a **lower triangular matrix.**

Example 12

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 1 & -2 & 0 \\ 5 & 5 & 4 \end{pmatrix}$$

Special case

- **Zero matrix**

3.7. Other Matrices

Each of the following matrices has its own separate chapter:

- **Transpose matrix** A^T
- **Inverse matrix** A^{-1}
- **Orthogonal matrix** Q
- **Rotation matrix** D