Adding Matrices

In this chapter we will look at how to add matrices.

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Required Prior Knowledge

Basics of Matrix Calculations

1. Requirement

Matrices can only be added if they have the same number of rows and columns.

Example 1

Is an addition of the matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$$

possible?

An addition of A and B is possible because both matrices have the same number of rows and columns.

Example 2

Is an addition of the matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

possible?

An addition of A and B is **not possible**, because the two matrices do not have the same number of rows and columns.

2. Sum Matrix

The result of the addition is called the **sum matrix**.

The sum matrix has exactly as many rows and columns as the matrices A and B.

3. Instructions

$$A+B=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}+\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}=\begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{pmatrix}$$

Matrices are added by adding the corresponding entries of the original matrices.

!!!

4. Example

Example 3

Add the matrices

$$A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \text{and} \quad B=\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}.$$

$$A+B=\begin{pmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{pmatrix}=\begin{pmatrix} 6 & 8 \\ 10 & 12 \end{pmatrix}$$

5. Calculation Rules

$$A+B=B+A$$

Commutative Law

$$(A+B)+C=A+(B+C)$$

Associative Law

Subtracting Matrices

In this chapter we will look at how to subtract matrices.

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Required Prior Knowledge

Basics of Matrix Calculations

1. Requirement

Matrices can only be subtracted if they have the same number of rows and columns.

Example 1

Is a subtraction of the matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix}$$

possible?

A subtraction of A and B is possible because both matrices have the same number of rows and columns.

Example 2

Is a subtraction of the matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

possible?

A subtraction of A and B is **not possible**, because the two matrices do not have the same number of rows and columns.

2. Difference Matrix

The result of the subtraction is called the **difference matrix**.

The difference matrix has exactly as many rows and columns as the matrices A and B.

3. Instructions

$$A - B = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} - \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} \\ a_{21} - b_{21} & a_{22} - b_{22} \end{pmatrix}$$

Matrices are subtracted by subtracting the corresponding entries of the original matrices.

Matrix Multiplication

In this chapter we learn how to multiply matrices.

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Required Prior Knowledge

- Basics of matrix calculations
- Calculating the dot product

1. Requirement

Two matrices can only be multiplied if the number of columns of the first matrix matches the number of rows of the second matrix.

Example 1

Is a multiplication of the matrices

$$A_{(2,3)} \cdot B_{(3,2)} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix}$$

possible?

Yes, multiplying A and B is possible because the number of columns of A matches the number of rows of B.

Example 2

Is a multiplication of the matrices

$$A_{(2,3)} \cdot B_{(2,2)} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

possible?

Multiplying A and B is **not** possible because the number of columns of A does not match the number of rows of B.

2. Product Matrix

The result of the multiplication is called the **product matrix**, **matrix product**, or **matrix multiplication result**.
The product matrix has as many rows as matrix A and as many columns as matrix B.

Example 3

$$A_{(2,3)} \cdot B_{(3,2)} = C_{(2,2)}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

Example 4

$$A_{(2,3)} \cdot B_{(3,4)} = C_{(2,4)}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} \cdot \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \end{pmatrix}$$

3. Calculation Rules

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

Associative Law

$$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$$

$$(A + B) \cdot C = (A \cdot C) + (B \cdot C)$$

Distributive Law

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WARNING!

In general:

$$A \cdot B \neq B \cdot A$$

The **commutative law** (of multiplication) does **not** apply to matrices!

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4. Matrix Multiplication Using the Falk Scheme

To multiply matrices by hand, the so-called “**Falk Scheme**” is commonly used.

1. Draw a cross
2. Enter matrix A in the lower-left
3. Enter matrix B in the upper-right
4. Enter the result matrix C in the lower-right
5. Calculate the elements of the result matrix C
6. Write down the result

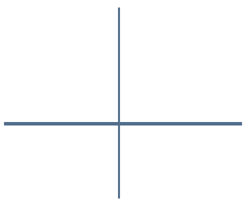
Example 5

Given the matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} \quad \text{und} \quad B = \begin{pmatrix} 2 & 1 \\ 1 & 2 \\ 2 & 1 \end{pmatrix}$$

Calculate the matrix product $A \cdot B$.

1. Draw a cross



2. Enter matrix A in the lower-left

1	2	3		
3	1	1		

3. Enter matrix B in the upper-right

			2	1
			1	2
			2	1
1	2	3		
3	1	1		

4 Enter the result matrix C in the bottom right

Note: You can skip this step if you have already solved some exercises.

			2	1
			1	2
			2	1
1	2	3	x_{11}	x_{12}
3	1	1	x_{21}	x_{22}

5 Calculate the elements of the result matrix C

x_{11} is obtained from the dot product of the 1st row of matrix A and the 1st column of matrix B.

			2	1
			1	2
			2	1
1	2	3	x_{11}	x_{12}
3	1	1	x_{21}	x_{22}

$$x_{11} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 1 \cdot 2 + 2 \cdot 1 + 3 \cdot 2 = 10$$

x_{12} is obtained from the dot product of the 1st row of matrix A and the 2nd column of matrix B.

			2	1
			1	2
			2	1
1	2	3	x_{11}	x_{12}
3	1	1	x_{21}	x_{22}

$$x_{12} = \begin{pmatrix}1\\2\\3\end{pmatrix} \cdot \begin{pmatrix}1\\2\\1\end{pmatrix} = 1 \cdot 1 + 2 \cdot 2 + 3 \cdot 1 = 8$$

and so on

			2	1
			1	2
			2	1
1	2	3	x_{11}	x_{12}
3	1	1	x_{21}	x_{22}

$$x_{21} = \begin{pmatrix}3\\1\\1\end{pmatrix} \cdot \begin{pmatrix}2\\1\\2\end{pmatrix} = 3 \cdot 2 + 1 \cdot 1 + 1 \cdot 2 = 9$$

			2	1
			1	2
			2	1
1	2	3	x_{11}	x_{12}
3	1	1	x_{21}	x_{22}

$$x_{22} = \begin{pmatrix}3\\1\\1\end{pmatrix} \cdot \begin{pmatrix}1\\2\\1\end{pmatrix} = 3 \cdot 1 + 1 \cdot 2 + 1 \cdot 1 = 6$$

<u>Here is the result:</u>

			2	1
			1	2
			2	1
1	2	3	10	8
3	1	1	9	6

$$C = \begin{pmatrix}10 & 8\\9 & 6\end{pmatrix}$$