Transposed Matrix

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1. Prerequisite

There are no prerequisites. Any matrix can be transposed.

2. Definition

The **transpose of a matrix** A^T is obtained by **swapping the rows and columns** of matrix A.

3. Transposing a Matrix

All three methods discussed below lead to the same result.

Method 1

A matrix is transposed by **turning rows into columns.**

The **1st row** of matrix A becomes the **1st column** of the transposed matrix A^T, and so on.

Example 1

$$A = \begin{pmatrix} 2 & 3 & 0 \\ 1 & 4 & 5 \end{pmatrix} \quad \to \quad A^T = \begin{pmatrix} 2 & 1 \\ 3 & 4 \\ 0 & 5 \end{pmatrix}$$

Method 2

A matrix is transposed by **turning columns into rows.**

The **1st column** of matrix A becomes the **1st row** of the transposed matrix A^T, and so on.

Example 2

$$A = \begin{pmatrix} 2 & 3 & 0 \\ 1 & 4 & 5 \end{pmatrix} \quad \rightarrow \quad A^T = \begin{pmatrix} 2 & 1 \\ 3 & 4 \\ 0 & 5 \end{pmatrix}$$

Method 3

A matrix is transposed by **reflecting it across the main diagonal**.

Example 3

$$A = \begin{pmatrix} 2 & 3 & 0 \\ 1 & 4 & 5 \end{pmatrix} \quad \rightarrow \quad A^T = \begin{pmatrix} 2 & 1 \\ 3 & 4 \\ 0 & 5 \end{pmatrix}$$

Here, the main diagonal acts like a mirror: each entry a_{ij} is moved to position a_{ji}.

4. Calculation Rules

1. $(A^T)^T = A$
   Transposing a matrix twice returns the original matrix.
2. $(A + B)^T = A^T + B^T$
   The transpose of a sum of matrices equals the sum of the transposed matrices.
3. $(A \cdot B)^T = B^T \cdot A^T$
   The transpose of a matrix product equals the product of the transposed matrices in **reverse order** (!).

5. Symmetric Matrices

If **A** = **A**^{T}, then the matrix **A** is called **symmetric**.

6. Antisymmetric Matrices

If **A** = −**A**^{T}, then the matrix **A** is called **antisymmetric** or **skew-symmetric**.

Examples

Symmetric Matrix

A matrix is **symmetric** if $A = A^T$.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$

Its transpose is

$$A^T = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix} = A$$

Therefore, A is **symmetric**.

Antisymmetric Matrix

A matrix is **antisymmetric** (or **skew-symmetric**) if $A = -A^T$.

$$B = \begin{pmatrix} 0 & 2 & -1 \\ -2 & 0 & 4 \\ 1 & -4 & 0 \end{pmatrix}$$

Its transpose is

$$B^T = \begin{pmatrix} 0 & -2 & 1 \\ 2 & 0 & -4 \\ -1 & 4 & 0 \end{pmatrix}$$

and

$$-B^T = \begin{pmatrix} 0 & 2 & -1 \\ -2 & 0 & 4 \\ 1 & -4 & 0 \end{pmatrix} = B$$

Therefore, B is **antisymmetric** (or **skew-symmetric**).