Inverse Matrix

In this chapter, we will look at what an inverse matrix is.

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1. Introduction

If you multiply a number by its reciprocal, the result is always 1.

Let's start ♡

A few decades ago, the inverse matrix was also called the **"reciprocal matrix."** This term clearly reflects its relationship to the **reciprocal** of a number.

Example

$$A \cdot A^{-1} = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 2 & -2 \\ 0 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 2 \\ -1 & 2 & 4 \\ -1 & 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

The reciprocal of a number can be calculated relatively easily. Unfortunately, this is not the case for matrices. To compute the reciprocal of a matrix—that is, the **inverse matrix**—we need one of the methods described in the next section.

2. Calculating the Inverse Matrix

There are essentially two methods for calculating the inverse matrix:

- **Calculate the inverse matrix using the Gauss–Jordan algorithm.**
- **Calculate the inverse matrix using the adjugate (adjoint) matrix.**

Another (less commonly used) method is to **calculate the inverse matrix using Cramer's Rule.**

3. Conditions for the Existence of an Inverse

Only **square matrices** can have an inverse. However, **not every square matrix is invertible**.

> **A square matrix A is invertible if and only if**
>
> $$\det(A) \neq 0.$$

Matrices whose rows or columns are **linearly dependent** have a determinant of zero. Therefore, they **do not have an inverse matrix**.

3.1 Regular Matrices

> **A square matrix A whose inverse A^{-1} exists is called a *regular matrix*.**

3.2 Singular Matrices

> **A square matrix A whose inverse A^{-1} does not exist is called a *singular matrix*.**

4. Calculation Rules

Rule 1: Inverse of a Product

$$(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$$

The inverse of a matrix product is equal to the product of the inverses in **reverse order**.

Rule 2: Inverse of a Transpose

$$(A^T)^{-1} = (A^{-1})^T$$

The inverse of the transpose of a matrix is equal to the transpose of the inverse matrix.

Rule 3: Inverse of an Inverse

$$(A^{-1})^{-1} = A$$

The inverse of a matrix is itself invertible.

The inverse of the inverse matrix is the original matrix.

Rule 4: Inverse of a Scalar Multiple

$$(k \cdot A)^{-1} = k^{-1} \cdot A^{-1}, \qquad k \neq 0$$

The inverse of a matrix multiplied by a scalar $k \neq 0$ is equal to the inverse of the matrix multiplied by the reciprocal of the scalar.