

INVERSE MATRIX EXAMPLE (Gauss-Jordan)

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Instructions

1. Set up the augmented matrix $(A \mid E)$
2. Transform matrix A in the augmented matrix into the identity matrix
3. Read off the inverse matrix A^{-1}

1. Set up the augmented matrix

From matrix A and the identity matrix E , we form the augmented matrix $(A \mid E)$.

2. Transform matrix A into the identity matrix

Using the **Gauss-Jordan algorithm**, we transform the left block of the augmented matrix, i.e. matrix A , into the identity matrix. The necessary row operations are applied to the entire augmented matrix $(A \mid E)$.

3. Read off the inverse matrix

Once the left block of the augmented matrix has become the identity matrix, the right block is the inverse matrix A^{-1} .

Example

Calculate the inverse of the following matrix using the Gauss-Jordan algorithm:

$$A = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 2 & -2 \\ 0 & -1 & 1 \end{pmatrix}$$

1. Set up the augmented matrix $(A \mid E)$

To save space, we omit the round brackets:

$$\left(\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 1 & 2 & -2 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right)$$

2. Transform matrix A into the identity matrix

First:

$$I \div 2$$

gives

$$\left(\begin{array}{ccc|ccc} 1 & -0.5 & 0 & 0.5 & 0 & 0 \\ 1 & 2 & -2 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right)$$

Then:

$$II - I$$

$$\left(\begin{array}{ccc|ccc} 1 & -0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 2.5 & -2 & -0.5 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right)$$

Next:

$$II \div 2.5$$

$$\left(\begin{array}{ccc|ccc} 1 & -0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 1 & -0.8 & -0.2 & 0.4 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array} \right)$$

Then:

$$III + II$$

$$\left(\begin{array}{ccc|ccc} 1 & -0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 1 & -0.8 & -0.2 & 0.4 & 0 \\ 0 & 0 & 0.2 & -0.2 & 0.4 & 1 \end{array} \right)$$

Next:

$$III \div 0.2$$

$$\left(\begin{array}{ccc|ccc} 1 & -0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 1 & -0.8 & -0.2 & 0.4 & 0 \\ 0 & 0 & 1 & -1 & 2 & 5 \end{array} \right)$$

Then:

$$II + 0.8 \cdot III$$

$$\left(\begin{array}{ccc|ccc} 1 & -0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 4 \\ 0 & 0 & 1 & -1 & 2 & 5 \end{array} \right)$$

Finally:

$$I + 0.5 \cdot II$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -1 & 2 & 4 \\ 0 & 0 & 1 & -1 & 2 & 5 \end{array} \right)$$

Als Ergebnis erhalten wir:

3. Read off the inverse matrix A^{-1}

The right-hand block is the inverse matrix:

$$A^{-1} = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 2 & 4 \\ -1 & 2 & 5 \end{pmatrix}$$

Verification

Multiplying A by A^{-1} gives the identity matrix:

$$A \cdot A^{-1} = \begin{pmatrix} 2 & -1 & 0 \\ 1 & 2 & -2 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 2 \\ -1 & 2 & 4 \\ -1 & 2 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E$$