# DETERMINANTS

## Meaning and Applications

- **Solvability of systems of equations:** If the determinant is not equal to zero, $\det(A) \neq 0$, the corresponding system of linear equations has exactly one unique solution.
- **Invertibility:** A matrix can only be inverted if its determinant is not equal to zero.
- **Geometric scaling factor:** The absolute value of the determinant indicates by what factor areas ($2 \times 2$) or volumes ($3 \times 3$) change when transformed by the matrix. A negative sign additionally indicates a reflection or reversal of orientation.
- **Eigenvalues:** The determinant of a matrix is equal to the product of all its eigenvalues. Determinats Part 1

## Calculation

Depending on the dimension of the determinant, there are different methods for calculating it. For $2 \times 2$ and $3 \times 3$ determinants, there is a specific formula for each. For determinants with more than three rows and columns, the **Laplace expansion theorem** and the **Gaussian elimination algorithm** are suitable.

- Calculating $2 \times 2$ determinants
- Calculating $3 \times 3$ determinants
- Laplace expansion theorem
- Calculating determinants using the Gaussian elimination algorithm

# Properties

### 1. Transpose

$$|A| = |A^T|$$

**In words:** The determinant of a matrix and the determinant of its transpose are identical.

### 2. Interchanging rows or columns

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = - \begin{vmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

**In words:** If two rows (or two columns) of a matrix are interchanged, the sign of the determinant changes. If three rows (or three columns) are interchanged, the sign does not change.

### 3. Multiplication of a row or column by a number

$$\begin{vmatrix} \lambda a_1 & \lambda a_2 & \lambda a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} \lambda a_1 & a_2 & a_3 \\ \lambda b_1 & b_2 & b_3 \\ \lambda c_1 & c_2 & c_3 \end{vmatrix} = \lambda \cdot \det(D)$$

**In words:** If a row (or a column) of a matrix is multiplied by a number, the determinant is also multiplied by that number.

## Further Properties

$$|A \cdot B| = |A| \cdot |B|$$

**In words:** The determinant of the product of two matrices is equal to the product of their determinants.

A determinant is equal to **zero** if:

- a row/column consists entirely of zeros,
- two rows/columns are identical,
- one row/column is a linear combination of other rows/columns.

## Applications

- Solving systems of linear equations using **Cramer's Rule**
- Calculating the inverse matrix using **Cramer's Rule**
- Calculating the rank of a matrix using **adjugates**
- Calculating the inverse matrix using **adjugates**

The translation above covers all three pages of the uploaded PDF, including the mathematical formulas and bullet points.