

DETERMINANTS

Calculating Determinants

In this chapter, we will look at how to calculate a determinant.

Contents

1. Calculating a 2×2 determinant
2. Calculating a 3×3 determinant
3. Calculating an $n \times n$ determinant

Prerequisite knowledge

- What is a determinant?

1. Calculating a 2×2 Determinant

The formula for calculating a 2×2 determinant is:

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = a \cdot d - c \cdot b$$

Example 1

$$\begin{aligned} |A| &= \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} \\ &= 1 \cdot 4 - 3 \cdot 2 \\ &= 4 - 6 \\ &= -2 \end{aligned}$$

Example 2

$$\begin{aligned} |B| &= \begin{vmatrix} 3 & -2 \\ 6 & -4 \end{vmatrix} \\ &= 3 \cdot (-4) - 6 \cdot (-2) \\ &= -12 + 12 \\ &= 0 \end{aligned}$$

2. Calculating a 3×3 Determinant

The formula for calculating a 3×3 determinant is:

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$= a \cdot e \cdot i + b \cdot f \cdot g + c \cdot d \cdot h - g \cdot e \cdot c - h \cdot f \cdot a - i \cdot d \cdot b$$

Example 3

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$= 1 \cdot 5 \cdot 9 + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8 - 7 \cdot 5 \cdot 3 - 8 \cdot 6 \cdot 1 - 9 \cdot 4 \cdot 2$$

$$= 45 + 84 + 96 - 105 - 48 - 72$$

$$= 0$$

Example 4

$$|B| = \begin{vmatrix} 2 & 5 & 2 \\ 3 & -3 & 1 \\ 1 & 4 & -4 \end{vmatrix}$$

$$= 2 \cdot (-3) \cdot (-4) + 5 \cdot 1 \cdot 1 + 2 \cdot 3 \cdot 4 - 1 \cdot (-3) \cdot 2 - 4 \cdot 1 \cdot 2 - (-4) \cdot 3 \cdot 5$$

$$= 24 + 5 + 24 - (-6) - 8 - (-60)$$

$$= 111$$

! This formula is called „Rule of Sarrus“- 

3. Calculating an n×n Determinant

For larger determinants, there are essentially two methods:

- Laplace expansion theorem