2. Definition

In the following, we want to find out how the **slope of a curve** is defined.

But how do we do that?

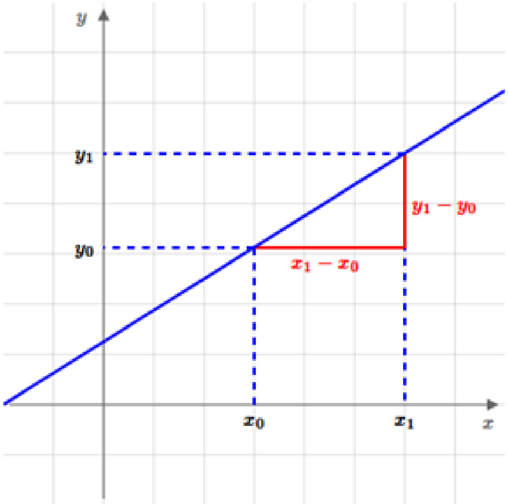
Idea

We apply the **slope triangle** to a curve!

We first discussed the **slope triangle** in the chapter on the slope of a linear function. It was used to derive the slope formula:

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

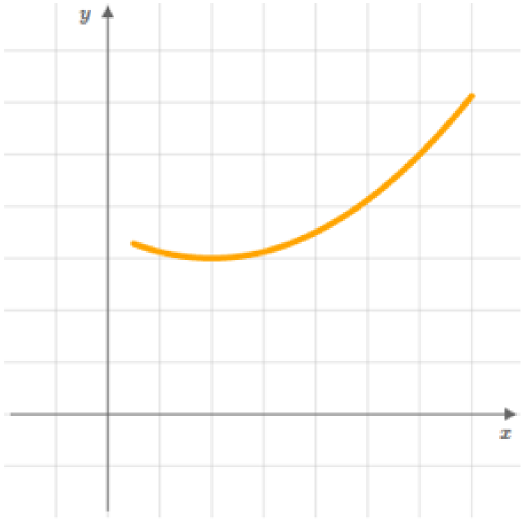
Here, m is the slope of a straight line.

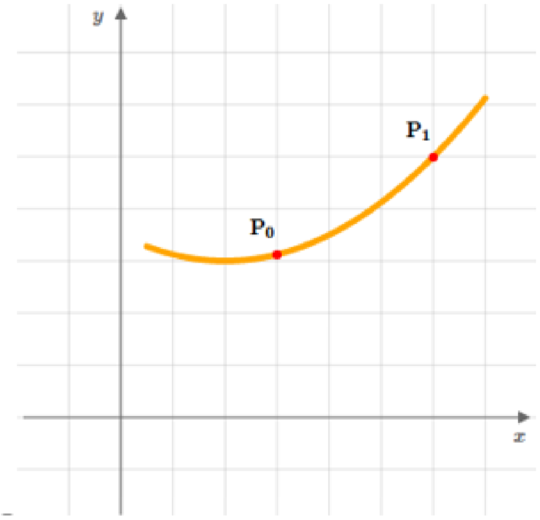


Applying the slope triangle to a curve

Now let us look at what happens when we apply the slope triangle to a curve.

First, we mark two arbitrary points.





The Secant

We draw a straight line through these two points.

A straight line that passes through two points of a curve is called a **secant**.

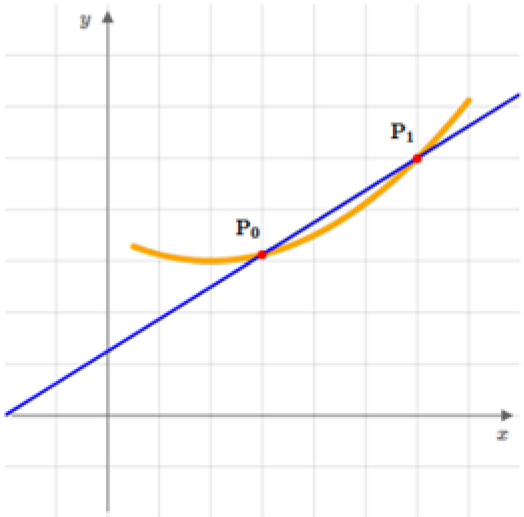
The formula for the slope of the secant can again be derived using the slope triangle.

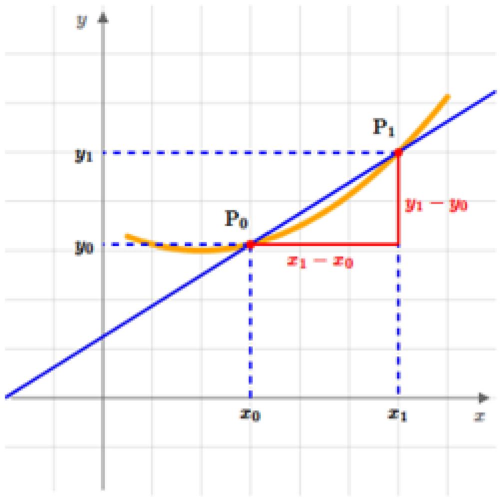
For the **slope of the secant** m, we therefore obtain:

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

This formula represents the **difference quotient** we are looking for.

The diagrams on pages 2 and 3 illustrate this construction: two points P_0 and P_1 are selected on the curve, and the straight line through them is the secant.





Difference Quotient

However, the following notation for the difference quotient is more commonly used:

$$y_1 = f(x_1)$$

and

$$y_0 = f(x_0).$$

The **difference quotient** is therefore:

$$\boxed{m = \frac{f(x_1) - f(x_0)}{x_1 - x_0}}$$

Difference Quotient

$$\boxed{m = \frac{f(x_1) - f(x_0)}{x_1 - x_0}}$$

The corresponding diagram shows the secant through the two points P_0 and P_1 on the curve, with the vertical difference

$$f(x_1) - f(x_0)$$

and the horizontal difference

$$x_1 - x_0.$$

3. Slope Formula vs. Difference Quotient

Slope Formula

$$m = \frac{y_1 - y_0}{x_1 - x_0}$$

Short notation:

$$m = \frac{\Delta y}{\Delta x}$$

Meaning:

$$m = \text{slope of a straight line}$$

The slope refers to the **entire straight line**.

Difference Quotient

$$m = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

Short notation:

$$m = \frac{\Delta y}{\Delta x}$$

Meaning:

$$m = \text{slope of the secant}$$

Here, the slope refers to the **secant of the curve**, i.e. the straight line that passes through the two points

$$P_0(x_0|y_0)$$

and

$$P_1(x_1|y_1).$$

Summary

1. $y_1 = f(x_1)$
2. $y_0 = f(x_0)$

$$\underbrace{m = \frac{y_1 - y_0}{x_1 - x_0}}_{\text{Steigungsformel}} \xrightarrow{y_1 = f(x_1) \text{ und } y_0 = f(x_0)} \underbrace{m = \frac{f(x_1) - f(x_0)}{x_1 - x_0}}_{\text{Differenzenquotient}}$$

3. $\Delta y = y_1 - y_0$
4. $\Delta x = x_1 - x_0$

$$m = \underbrace{\frac{y_1 - y_0}{x_1 - x_0}}_{\text{Steigungsformel}} = \underbrace{\frac{f(x_1) - f(x_0)}{x_1 - x_0}}_{\text{Differenzenquotient}} \xrightarrow[\Delta x = x_1 - x_0]{\Delta y = y_1 - y_0} \underbrace{m = \frac{\Delta y}{\Delta x}}_{\text{Abkürzung}}$$