

Table of Differentiation Rules in Differential Calculus

	Function	Derivative
Power Rule	$f(x) = x^n$	$f'(x) = n \cdot x^{n-1}$
Constant Function	$f(x) = k$	$f'(x) = 0$
Constant Multiple Rule	$f(x) = a \cdot g(x)$	$f'(x) = a \cdot g'(x)$
Sum Rule	$f(x) = g(x) + h(x)$	$f'(x) = g'(x) + h'(x)$
Product Rule	$f(x) = g(x) \cdot h(x)$	$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$
Quotient Rule	$f(x) = \frac{g(x)}{h(x)}$	$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h(x)^2}$
Chain Rule	$f(x) = g(h(x))$	$f'(x) = g'(h(x)) \cdot h'(x)$

Power Rule:

$$f(x) = x^n$$

$$f'(x) = n \cdot x^{n-1}$$

Example:

$$f(x) = 5 \cdot x^3$$

$$f'(x) = 3 \cdot 5 \cdot x^{3-1}$$

$$f'(x) = 15 \cdot x^2$$

Constant Faktor Rule

$$f(x) = a \cdot g(x) \quad \Rightarrow \quad f'(x) = a \cdot g'(x)$$

The constant factor rule simply states that when we have a constant factor in front of a function, we do not need to take it into account separately when differentiating. We can simply bring it along throughout the differentiation process and place it back in front of the derivative function.

Example:

$$f(x) = 3 \cdot x^2$$

$$f'(x) = 3 \cdot g'(x) = 3 \cdot (2x) = 6x.$$

Before starting with some „special“ Funktionen (sin(x) ; ln(x)) following a Table of First Derivatives of Important Functions

Function $f(x)$	Derivative $f'(x)$
c (constant)	0
x	1
x^n	nx^{n-1}
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}$
$\frac{1}{x}$	$-\frac{1}{x^2}$
e^x	e^x
a^x	$a^x \ln(a)$
$\ln(x)$	$\frac{1}{x}$
$\log_a(x)$	$\frac{1}{x \ln(a)}$
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$

Sum Rule (Differentiation)

$$f(x) = g(x) + h(x)$$

$$f'(x) = g'(x) + h'(x)$$

The sum rule makes differentiation much easier because we can work through the individual terms one at a time.

Example:

$$f(x) = 3x^2 + 2x + \sin(x)$$

Using the rule, we can now consider each term separately:

$$g(x) = 3x^2, \quad h(x) = 2x, \quad k(x) = \sin(x)$$

The derivatives of the individual terms are therefore:

$$g'(x) = 6x, \quad h'(x) = 2, \quad k'(x) = \cos(x)$$

Finally, we obtain the derivative of the function by adding all the individual derivatives:

$$f'(x) = 6x + 2 + \cos(x)$$

Product Rule (Differentiation)

The derivative of a product of two functions is:

$$f(x) = g(x) \cdot h(x)$$

$$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

Products can be split into their individual factors and differentiated separately.

Example:

Given the function:

$$f(x) = x^2 \cdot \sin(x)$$

We identify the two factors:

$$g(x) = x^2 \quad \text{and} \quad h(x) = \sin(x)$$

Differentiate each factor:

$$g'(x) = 2x \quad \text{and} \quad h'(x) = \cos(x)$$

Now apply the product rule:

$$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

Substitute the derivatives:

$$f'(x) = (2x) \sin(x) + x^2 \cos(x)$$

Final result

$$f'(x) = 2x \sin(x) + x^2 \cos(x)$$

Quotient Rule (Differentiation)

The quotient rule states:

$$f(x) = \frac{g(x)}{h(x)} \implies f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{h(x)^2}$$

A quotient can also be converted into a product, but the quotient rule saves us this step.

Example:

$$f(x) = \frac{\sin(x)}{x}$$

Thus, we have the numerator function

$$g(x) = \sin(x)$$

and the denominator function

$$h(x) = x$$

Differentiate the two functions:

$$g'(x) = \cos(x) \quad \text{and} \quad h'(x) = 1$$

Now apply the quotient rule:

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{h(x)^2}$$

Substituting the functions:

$$f'(x) = \frac{\cos(x) \cdot x - \sin(x) \cdot 1}{x^2}$$

Therefore:

$$f'(x) = \frac{x \cos(x) - \sin(x)}{x^2}$$

Chain Rule (Differentiation)

The chain rule states:

$$f(x) = g(h(x)) \implies f'(x) = g'(h(x)) \cdot h'(x)$$

The chain rule allows, among other things, the differentiation of expressions in parentheses or more complicated exponents.

Example:

$$f(x) = (4x^2 + 2)^2$$

We now have the so-called “**outer function**” represented by the parentheses, and the “**inner function**” represented by the contents of the parentheses.

$$f(x) = g(h(x)) \implies g(h(x)) = h(x)^2 \quad \text{and} \quad h(x) = (4x^2 + 2)$$

$$g'(h(x)) = 2 \cdot h(x) \quad \text{and} \quad h'(x) = 8x$$

Now apply the chain rule:

$$f'(x) = g'(h(x)) \cdot h'(x)$$

$$= 2 \cdot h(x) \cdot 8x$$

$$= 2 \cdot (4x^2 + 2) \cdot 8x$$

$$\boxed{f'(x) = 16x \cdot (4x^2 + 2)}$$