

Tangent to a Function

- a) From a given Point on the curve of the function
- b) Parallel to an existing Tangent
- c) From a given Point from anywhere outside the function

a)

Tangent to a Function

The equation of a tangent at the point $P(x_0|y_0)$ is determined using the general equation of a straight line $t(x) = m \cdot x + b$, or the direct tangent formula.

Step-by-step procedure

- **Determine the point of tangency:** Substitute the given value x_0 into the original function $f(x)$ to obtain the corresponding y -value $y_0 = f(x_0)$. You then have the complete point $P(x_0|y_0)$.
- **Find the derivative:** Calculate the first derivative $f'(x)$ of the function.
- **Calculate the slope:** Substitute the x -value x_0 into the derivative:

$$m = f'(x_0)$$

This is the slope m of the tangent.

- **Determine the y-intercept:** Substitute the slope m and the coordinates of the point $P(x_0|y_0)$ into the linear equation

$$y = m \cdot x + b$$

and solve for b :

$$b = y_0 - m \cdot x_0$$

Alternatively, you can use the direct formula:

$$t(x) = f'(x_0) \cdot (x - x_0) + f(x_0)$$

b)

Tangent Parallel to an Existing Tangent

If you want to determine a **tangent to a function that is parallel to an existing tangent**, proceed as follows:

Assume that the known tangent touches the function at x_0 . Its slope is

$$m = f'(x_0)$$

For a **parallel tangent**, the slope must be identical. Therefore, we are looking for another point x_1 such that

$$f'(x_1) = f'(x_0)$$

Procedure

1. Determine the slope of the existing tangent

$$m = f'(x_0)$$

2. Set the derivative equal to this slope

$$f'(x) = m$$

3. Solve for x

The solutions $x_1, x_2, \dots$ are the possible points of tangency for the parallel tangents.

4. Determine the corresponding point

$$y_1 = f(x_1)$$

5. Determine the tangent equation

$$t(x) = m(x - x_1) + f(x_1)$$

Example

Given:

$$f(x) = x^2$$

The existing tangent is at $x_0 = 1$.

Derivative:

$$f'(x) = 2x$$

Slope of the existing tangent:

$$m = f'(1) = 2$$

For a parallel tangent:

$$f'(x) = 2$$

Therefore:

$$2x = 2 \Rightarrow x = 1$$

There is **no second parallel tangent** because the parabola x^2 has the same slope at no other point.

c)

Tangent Line Through an Arbitrary Point Outside the Function

Step-by-Step Procedure

1. Find the derivative

Calculate the first derivative $f'(x)$ of the given function.

2. Substitute the point

Substitute x_0 and y_0 — the coordinates of the given external point P — into the general tangent-line equation.

Also express $f(u)$ and $f'(u)$ in terms of the variable u .

3. Solve for the point of tangency

Solve the resulting equation for u .

Note: There may be no solution, one solution, or several solutions for u .

4. Finalize the tangent-line equation

Substitute the value or values found for u back into the general tangent-line equation and convert it to the final form

$$y = mx + b.$$

Step-by-Step Example

Given the parabola

$$f(x) = x^2 - 1$$

and the external point

$$P(3 \mid 4).$$

Step 1: Determine the derivative

$$f'(x) = 2x$$

therefore

$$f'(u) = 2u.$$

Steps 2 & 3: Substitute the coordinates of $P(3 \mid 4)$ and solve for u

We use the formula

$$y = f'(u) \cdot (x_0 - u) + f(u).$$

Substituting $x_0 = 3$ and $y = 4$:

$$4 = 2u \cdot (3 - u) + (u^2 - 1)$$

$$4 = 6u - 2u^2 + u^2 - 1$$

$$4 = -u^2 + 6u - 1$$

$$0 = -u^2 + 6u - 5$$

or

$$u^2 - 6u + 5 = 0.$$

Using the quadratic formula, we obtain two points of tangency:

$$u_1 = 1$$

and

$$u_2 = 5.$$

Step 4: Determine the Tangent-Line Equations

We now substitute the values of u one after another into the general formula.

For $u_1 = 1$:

$$f(1) = 1^2 - 1 = 0$$

and

$$f'(1) = 2 \cdot 1 = 2.$$

Thus,

$$y = 2 \cdot (x - 1) + 0$$

and therefore

$$\boxed{y = 2x - 2}.$$

For $u_2 = 5$:

$$f(5) = 5^2 - 1 = 24$$

and

$$f'(5) = 2 \cdot 5 = 10.$$

Thus,

$$y = 10 \cdot (x - 5) + 24$$

and therefore

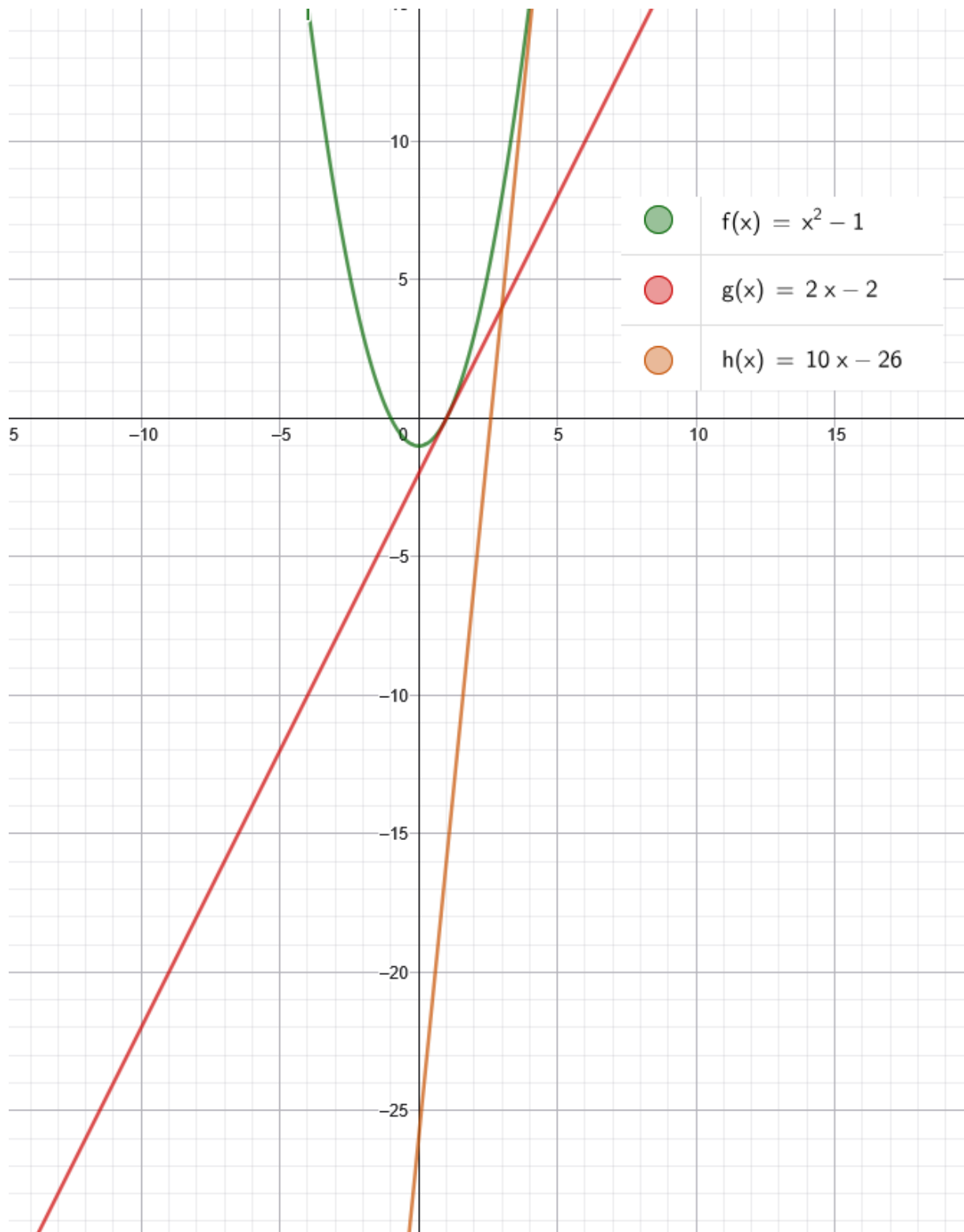
$$\boxed{y = 10x - 26}.$$

Result: The external point $P(3, 4)$ has **two tangent lines** to the parabola $f(x) = x^2 - 1$:

$$\boxed{y = 2x - 2}$$

and

$$\boxed{y = 10x - 26}.$$



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